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NONPARAMETRIC BAYES ESTIMATION OF DISTRIBUTION FUNCTIONS AND TH--ETC(U)
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In work under this contract, major results were obtained in the broad areas of survival analysis and life testing, probability density estimates and laws of large numbers, estimation after testing, robustness and distribution-free procedures, and stochastic systems. In particular, nonparametric esti-

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mators of the failure rate function and survival probability were developed under the assumption of increasing failure rate using both maximum likelihood and Bayesian approaches. These particular results have attracted wide attention due to their generality and applicability in survival analysis and reliability estimation from arbitrarily right-censored data. Also, consistency results for both univariate and multivariate kernel estimates for probability density functions and regression functions were obtained using techniques and results of function-space probability theory. Stochastic convergence results for weighted sums of random elements in various function spaces were also obtained. Sequential procedures were developed and analyzed which provided interval estimators of the parameter of interest after testing certain hypotheses. Various robustness and nonparametric methods for incomplete samples and broken samples were studied, as well as nonparametric repeated significance testing and randomized block designs. Thus, maintenance policies and development of new, more reliable, equipment may be formulated using the statistical procedures and theory from these results.

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I. Introduction

This document reports in detail the work performed and other research activities of the principal investigators during the period from June 1, 1979, to May 31, 1981, under contract number [redacted]-F49620-79-C-0140.

In Section 2 a comprehensive statement of the research objectives during this period is given, and in Section 3 a detailed report of the accomplishments and highlights of the significant results obtained is presented. Section 4 contains a complete listing of approximately thirty-one publications which have resulted from these research efforts. The personnel supported under this contract are listed in Section 5, and a list of the professional interactions--talks at conferences and meetings, colloquia and seminars, etc.-- is presented in Section 6. Section 7 gives an indication of the significance of the results to military application. Finally, other professional activities of the principal investigators are outlined in Section 8.

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2. Research Objectives During the Contract Period

The research objectives of this contract can be divided into five broad catagories or areas:

(A) Survival analysis and life testing;

(B) Probability density estimates and laws of large numbers;

(C) Estimation after testing;

(D) Robustness and distribution-free procedures;

(E) Stochastic systems theory.

The research problems which were considered under this contract in these five areas will be outlined in this section. The progress toward their solution and significant results will be described in Section 3 of this report.

A. Survival Analysis and Life Testing

The main thrust of the research in this area was to develop estimators of survival probability or reliability in nonparametric settings or in a nonparametric Bayesian framework which were much better than existing estimators in the sense of their smoothness and convergence properties. The main objective in this area was to develop such estimators for arbitrarily right-censored observations under no distributional assumptions other than assumptions concerning the failure rate. In particular, since increasing failure rate (IFR) distributions are very commonly used to model wear-out failures, the distributional assumption was simply that the underlying life time distribution was IFR. One of the most promising approaches to these problems was to consider (nonparametric) maximum likelihood estimation of an increasing failure rate function $r(t)$, $t \geq 0$, based on arbitrarily right-censored observations from the life distribution F and then to estimate the survival probability

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$\bar{F}(t) = 1 - F(t) = \exp(-\int_0^t r(u)du)$ from the maximum likelihood estimate of r. The other promising avenue of attack for these problems was to assume a prior distribution on the space of all increasing failure rate functions and obtain a nonparametric Bayes estimator of F or $\bar{F}$ which was a continuous function. A type of Poisson "shock model" was successfully used as a prior distribution.

Another objective in this area was to develop powerful nonparametric tests for symmetry of a bivariate distribution function with observations which are subject to arbitrary right censorship. The usual sign test is not the best test to use in the case of censoring since a pair of observations, one of which is censored, may not be comparable.

Also, under the parametric inverse Gaussian and Birnbaum-Saunders models, it was desired to obtain Bayes estimators of the reliability function. Approximate prediction intervals for the mean of future lifetimes from the inverse Gaussian model were to be investigated, as well as the problem of estimating reliability under growth assumptions for the two-parameter gamma distribution.

B. Probability Density Estimates and Laws of Large Numbers

This area of the research concentrated on obtaining consistency of probability density estimates and developing laws of large numbers which would have applications in the estimation of probability density functions. In particular, the kernel density estimation techniques were examined with the research objective of obtaining a wider class of kernel functions and more general bandwidth sequences which could be used while preserving the stochastic convergence of the estimate to the true underlying probability density function. The development of multivariate kernel density estimates

with convergence properties which were similar to the univariate estimates was also a research objective.

In the kernel density estimation, a kernel function (usually a probability density function which the experimenter may choose) is centered at each observed sample point and is scaled, depending on the sample size, so that its spread decreases and becomes more concentrated around the sample observation as the sample size increases. The estimate is then an average of these kernel functions which are centered at the sample observations. Unfortunately, the changing bandwidth (as the sample size increases) forces consideration of a Marcinkiewicz-Zygmund type of such an average. Hence, laws of large numbers in function spaces containing the possible kernel functions would yield consistency of these estimates and was a major research objective of the grant. Since the scaled kernel functions changed as the sample size changed, the appropriate formulation involved weighted sums of arrays of function-valued random variables. Also, the possible different measures of dispersion of the kernel estimate $f_n(t)$ from the probability density function $f(t)$ necessitated the consideration of laws of large numbers for Banach spaces and the function space $D[0,1]$ as a research objective.

C. Estimation After Testing

Suppose F_θ is a distribution function which is stochastically nondecreasing in θ. It was desired to obtain various sequential procedures which allow quick rejection of the null hypothesis $H_o: \theta \geq \theta_o$ when the alternative $H_1: \theta \leq \theta_1$ $(\theta_o > \theta_1)$ is true. In addition, these procedures would provide an accurate interval estimate of the parameter θ when the null hypothesis is accepted. The results would hold in particular for an exponential distribution with mean θ.

D. Robustness and Distribution-free Procedures

Several of the research objectives were in this category. One is concerned with the distributional robustness when there are missing observations in the samples. The second one was to establish the robustness (asymptotic conservativeness) of a K-sample Kruskal-Wallis test under certain departures from mutual independence of K univariate samples. Another objective was to investigate an asymptotically distribution-free simultaneous confidence region of treatment differences in a randomized complete block design. Also, it was desired to obtain nonparametric repeated significance tests for two-sample survival data to test equality of the two distributions and to estimate location difference for fragmentary samples. The estimation of the ratio of the scale parameters of two distributions based on arbitrarily right-censored observations from each distribution was also to be considered in detail. Very good results were obtained in all of these areas of investigation.

E. Stochastic Systems Theory

The main objective in the study of stochastic systems was to obtain very general conditions for the existence and stability of solutions to a class of stochastic systems which can be described by stochastic integral equations involving McShane integrals. This class includes stochastic integro-differential systems. Frequency domain conditions for the stability of the solution were to be considered.

3. Results of the Research Effort

In this section, a substantive summary of the significant accomplishments towards achieving the research objectives of this contract as outlined in Section 2 of this report will be given. The specific research papers containing the results will be listed in Section 4. Copies of these research papers have been forwarded to the Program Manager as they were submitted for publication.

A. Survival Analysis and Life Testing

One of the most significant results in this area is paper number 1 in the listing of Section 4. This paper had received wide attention from workers in this area, even before it had appeared in print, since it was presented at the annual Institute of Mathematical Statistics Meeting in Washinton, D.C., in August, 1979. The result concerns the (nonparametric) maximum likelihood estimation of an increasing failure rate function and the corresponding distribution function based on a set of observations subject to arbitrary right censorship. The estimator is developed using the techniques of isotonic regression, is defined everywhere on the positive real line, is a smooth function, and decreases to zero as time increas The small sample properties of the estimator were compared with the Kaplan-Meier estimator by a Monte Carlo study for Weibull distributions, and the results indicated that the maximum likelihood estimator is superior to the Kaplan-Meier estimator. This estimator is applicable to a broad spectrum of problems due to the generality of the censoring mechanism. For example, medical trial data as well as industrial life test data may be analysed with this estimation procedure. In particular, since no assumptions are required concerning the exact lifetime distribution, this estimator is especially applicable to the assessment of military equipment reliability and maintenance. Such equipment which is subject to wear-out may be assumed to follow an IFR life distribution, and this is the only assumption necessary for the estimation procedure developed here.

A second significant result in this area is contained in paper number 5 in Section 4. Bayesian nonparametric estimators of the survival function, the failure rate function, and the density function are obtained using Poisson jump processes to generate prior probability measures on the space of increasing failure rate functions. The jump processes are intuitively appealing and have a meaningful physical interpretation as a shock model, where "shocks" occurring to the system increase the failure rate by a positive amount at the random instant of occurrence. The Bayesian nonparametric estimators are also easily derived for these priors for arbitrarily right-censored observations. These estimators are continuous and are thus more appealing in this sense than estimators of other authors who used Dirichlet processes as prior distributions for (discrete with probability one) distributions. The same comments made on the applicability of the estimator in the last paragraph can be made for these nonparametric Bayes estimators. Also, along these same lines when the Dirichlet process is not completely specified, several sequences of nonparametric empirical Bayes estimators of survival probability were considered (paper number 18 in Section 4) and their asymptotic optimality with respect to a Dirichlet process was investigated.

In testing a bivariate life distribution for symmetry based on arbitrarily right-censored observations, a conditionally distribution-free test was developed(paper number 20 in Section 4). A permutation test using Gehan's scores is proposed which is much more powerful than the sign test. The power of the test was compared with that of the sign test by computer simulations using the Marshall-Olkin bivariate exponential model.

For the inverse Gaussian distribution as a parametric lifetime model, Bayes estimators of the reliability function have been obtained (paper number 12 in Section 4). For the case that the mean lifetime is known, Bayes estimators

of the reliability are obtained with respect to squared error loss for Jeffreys' noninformative prior and for the gamma family (natural conjugate family) of priors for the scale parameter. In the case that both the mean and scale parameter are unknown, an estimator of reliability is suggested which is based on the Bayes estimator for the first case. This estimator compares favorably with the maximum likelihood and minimum variance unbiased estimators.

The Birnbaum-Saunders fatigue life distribution with shape parameter α and scale parameter β was also considered (paper number 19 in Section 4). The scale parameter β is also the median lifetime, and assuming that β was known, Bayes estimators of the reliability function were obtained for a family of proper conjugate priors as well as for Jeffreys' vague prior for α. When both α and β were unknown, a modified Bayes estimator $R_B^*(t)$ of the reliability function was proposed using a moment estimator of β. In addition to being computationally simpler than the maximum likelihood estimator of the reliability function, Monte Carlo simulations for small samples showed that $R_B^*(t)$ was better than the method of moments estimator for all α and as good as the maximum likelihood estimator for small α in the sense of mean square errors.

The problem of predicting, on the basis of an observed sample from an inverse Gaussian distribution, the mean of a future random sample from the same distribution was also considered (paper number 13 in Section 4). Approximate prediction intervals were developed since it seemed to be impossible to derive exact intervals. The accuracy of the approximate intervals was investigated by extensive Monte Carlo simulations and found to be very good. These results are useful for predicting the next first passage time

for a Brownian motion with positive drift or the failure time of an item having inverse Gaussian life distribution.

Finally, in this area the estimation of the reliability function under development testing was investigated (paper number 22 in Section 4). Changes are often made at various stages during the development of a system to correct design weaknesses. This results in reliability growth, an increase in reliability and mean life of such systems as development continues. Assuming that the failure distribution at stage i of K development stages is gamma with parameters α_i and β_i, approximate maximum likelihood estimates of α_i, β_i, $i=1,\ldots,K$, were obtained subject to the conditions $\alpha_1 \le \ldots \le \alpha_K$ and $\beta_1 \le \ldots \le \beta_K$, which imply reliability growth. An iterative procedure involving two constrained nonlinear optimization problems was presented for obtaining the approximate maximum likelihood estimates. The solutions to the constrained optimization problems were relatively simple to compute using techniques of isotonic regression, and the iterations tended to converge rather quickly.

B. Probability Density Estimators and Laws of Large Numbers

In paper number 2 in the listing of Section 4, the uniform consistency of the multivariate kernel density estimate, $\hat{f}_n(x_1,\ldots,x_k)$, was obtained by showing that $||\hat{f}_n(x_1,\ldots,x_k) - f(x_1,\ldots,x_k)||_\infty$ converges completely to zero for a large class of weight functions and under mild conditions on the bandwidth sequences $b_k(n)$'s. This significant result provided for the uniform consistency of the estimate $\hat{m}_n(x_1,\ldots,x_k) = \dfrac{\hat{h}_n(x_1,\ldots,x_k)}{\hat{f}_n(x_1,\ldots,x_k)}$ for the regression function

$m(x_1,\ldots,x_k) = E(Y|X_1 = x_1,\ldots,X_k = x_k)$ where $f(x_1,\ldots,x_k) = \int f^*(x_1,\ldots,x_k,y)\,dy$ and $h(x_1,\ldots,x_k) = \int y f^*(x_1,\ldots,x_k,y)\,dy$. In these results the rates of convergence need not be uniform in each coordinate and the broad class of possible

kernels included Epanechnikov's optimal kernel and was characterized by a smoothness condition.

In papers number 24 and 25 in the listing of Section 4, the stochastic convergence of weighted sums of arrays of random elements in Banach spaces was obtained under varying conditions. One striking result was that the convergence of these weighted sums was necessary and sufficient for the geometric property, type p, of the Banach space. As corollaries, these results have Banach space versions of several random variables results. In paper number 9 it is shown in the application of these results that the consistency in the L^1-norm of certain density estimates is sufficiently implied by $\sum_{n=1}^{\infty}(b_n^2 n)^{-q} < \infty$ for some $q > 1$ where n is the sample size and b_n is the bandwidth. The L^p-consistency for a broad class of kernel estimates is obtained from the results of paper number 24 when $\sum_{n=1}^{\infty}(nb_n)^{-q} < \infty$ for some $q > 0$. Hence, the bandwidth sequence b_n could be of order $n^{-\delta}$ for any $0 < \delta < 1$. This significant improvement for general classes of kernel functions and density functions (including multivariate considerations) is mainly due to the probability cancellation in type p spaces and the L^p norm.

Let $\{w_n\}$ be a sequence of positive constants and $W_n = w_1+\ldots+w_n$ where $W_n \to \infty$ and $W_n/w_n \to \infty$. Let $\{X_n\}$ be a sequence of independent random elements in $D[0,1]$. In paper number 11 the almost sure convergence of $W_n^{-1} \sum_{k=1}^{n} w_k X_k$ is established under integral conditions and growth conditions on the weights $\{w_n\}$. These results are shown to be substantially stronger than the weighted sums convergence results of Taylor and Daffer (1980), J. Mult. Anal., and the strong laws of large numbers of Ranga Rao (1963), Theory of Probab. & Appl.'s, and Daffer and Taylor (1979), Annals of Probab.

In paper number 26 various integral conditions are examined with respect to the convergence almost surely and in probability of the general linear form $\sum_{k=1}^{\infty} a_{nk} X_k$ where $\{X_k\}$ are random elements in D[0,1] and $\{a_{nk}\}$ is an array of real numbers. Since the hypotheses of the results of this paper are in general much less restrictive than tightness (or convex tightness), these results represent significant improvements over existing laws of large numbers and convergence results for weighted sums of random elements in D[0,1]. Also, a counterexample is provided for both the strong and weak laws of large numbers when the hypotheses are weakened.

Weak laws of large numbers in Banach spaces have typically been for orthogonal or independent random elements and have not been related to the geometric properties of the space. In Howell and Taylor (paper number 6), a class of dependent random elements were studied, and weak laws of large numbers for this class (unconditional semibasic random elements) were obtained where the convergence rates depend on the geometric structure of the Banach space. In particular, convergence in probability of $\sum_{k=1}^{n} a_{nk} X_{nk}$ is obtained for random elements $\{X_{nk}\}$ satisfying various distributional conditions, including independence, conditional independence, and unconditional semi-basic, and weights $\{a_{nk}\}$ such that $\sum_{k=1}^{n} |a_{nk}|^p \le 1$ for each n where $1 \le p \le 2$ is related to the geometric Rademacher type of the space. Where previous results required type $p+\delta$ for the space, these results indicate type p is sufficient. In addition, these results include a larger class of random elements than just independent and conditionally independent. Since $\sum_{k=1}^{n} |a_{nk}|^p \le 1$ with $1 \le p \le 2$ leads naturally to applications to kernel density estimates, let

$$\frac{1}{nh_n} \sum_{k=1}^{n} K\left(\frac{t-X_k}{h_n}\right) = \sum_{k=1}^{n} a_{nk} X_{nk} .$$

In particular, the use of dependent arrays of random elements and the convergence of weighted sums is more appropriate for the sequential kernel density estimates where the bandwidth sequence may depend on the observations $X_1,\ldots,X_{n-1}$, that is, $h_n \equiv h_n(X_1,\ldots,X_{n-1})$.

In paper number 7 the weights are again restricted to be of the form $a_{nk} = w_k/W_n$ and the almost sure convergence of $\frac{1}{W_n}\sum_{k=1}^{n} w_k X_k$ was obtained under varying moment and distribution conditions on $\{X_k\}$. In particular, if the Banach space is of type p and $\{X_k\}$ have uniformly bounded tail probabilities by a real random variable X, then

$$\int_0^\infty t^{p-1} P[X > t] \int_t^\infty \frac{N(y)}{y^{p+1}}\, dydt < \infty \qquad (*)$$

is sufficient for the convergence where $N(y) = \text{card}\{n: W_n/w_n \le y\}$. Examination of (*) in terms of moment conditions on $\{X_k\}$ and growth conditions on the weights $\{w_n\}$ was included. In the absence of geometric conditions on the Banach space, tightness and the integral condition in (*) are used to obtain the almost sure convergence of the linear forms when $W_n = w_1+\ldots+w_n$. These results extend both the real-valued and Banach space results of previous papers by several authors.

C. Estimation After Testing

In the quality control of industrial or military production, it is common practice to test hypotheses to make a decision about acceptance or rejection of a production batch. Generally, a sequential testing procedure requires less data to reach a decision than a fixed sample or fixed time testing procedure. Also, in many situations, the experimenter may want to obtain a point or an interval estimate of some parameter after a test of hypothesis has been established.

Let F_θ be a distribution function which is stochastically nondecreasing in θ. In particular, F_θ can be an exponential distribution with mean θ. Several sequential procedures were developed which allow quick rejection of the null hypothesis $H_o : \theta \geq \theta_o$ when the alternative $H_1 : \theta \leq \theta_1 (\theta_1 < \theta_o)$ is true under the general model F_θ. In addition, these results provide accurate interval estimators of the parameter θ after the test has been completed (papers 14, 21).

D. Robustness and Distribution-free Procedures

First, a simple and robust estimator of the difference of location parameters of correlated variables is proposed when some observations on either of the variables are missing. We show that this estimator is consistent, asymptotically normally distributed, and insensitive to outlying observations. Asymptotic relative efficiency comparisons with other known estimators are made to show the advantage of the proposed estimator (paper 9).

Secondly, the robustness (asymptotic conservativeness) of Kruskal-Wallis test under certain departures from mutual independence of K univariate samples is established (paper 10). This robustness provides a procedure for testing the equality of K marginal distribution frunctions based on a broken random sample from a K-variate distribution which satisfied a mild condition. For the unbroken sample, a generalized Kruskal-Wallis test is proposed for testing the symmetry of a K-dimensional distribution function. The relative efficiency of the K-W test against the aligned rank order test is also examined under the normal shift model.

Thirdly, for a randomized complete block design with additive block effects, an asymptotically distribution-free simultaneous confidence region

of pairwise treatment differences is presented. The corresponding confidence bound has an explicit form and is easily obtained. Asymptotic relative efficiency comparisons with other known confidence regions were made to show the advantage of the proposed procedure. An example is provided for illustration purpose. The case of treatment against control is also discussed (papers 27 and 28).

Also, the problem of nonparametric two-sample repeated significance tests was studied (paper number 30). The asymptotic distribution theory of sequentially computed modified Wilcoxon scores was developed for two-sample survival data with random staggered entry and random loss to follow-up. A repeated significance testing procedure was presented for testing the equality of two survival distributions based on the asymptotic theory. The early stopping properties of this procedure was illustrated by an example.

An asymptotically distribution-free confidence interval for location difference of two correlated variables with incomplete data on both responses was obtained in paper number 17 of Section 4. The confidence limit has an explicit form and can be easily obtained. The new procedure was also shown to be more robust than other known parametric interval estimators.

In paper number 15 the point estimation of the ratio of scale parameters in the two-sample problem with arbitrarily right-censored observations was considered from the minimum distance approach. The two-sample Cramer-von Mises distance was used. The estimator was shown to be strongly consistent and small sample properties were exhibited by computer simulation results. Also, in paper 31 nonparametric point and interval estimators of the ratio of two scale parameters were given for arbitrarily right-censored data based on the idea of Hodges and Lehmann. These estimators were defined in terms of rank

test statistics for testing the equality of two survival distributions. The asymptotic properties and efficiencies of estimators corresponding to the Gehan-Gilbert test and to the logrank test were investigated.

Finally in this area, paper number 16 was a review article on Friedman's urn model written for the Encyclopedia of Statistical Science. This urn model has been shown to be very useful in the design of sequential experiments.

E. Stochastic Systems Theory

In paper number 8 of Section 4, general stochastic integral systems involving McShane stochastic integrals were studied. These general systems included classes of stochastic integro-differential systems. Existence, uniqueness, and stability of solutions to such systems were investigated using contractor theory, which is a generalization of the contraction principle for nonlinear operators. In particular, the results yielded frequency-type conditions for the stability of a class of stochastic systems.

4. Cumulative List of Written Publications in Technical Journals

In this section, the research papers which have been written under this contract are listed. They are divided into three categories: Published, accepted for publication, and submitted. Copies of the papers have already been forwarded to the Program Manager as they were submitted for publication or appeared in print, and sufficient copies of reprints will be forwarded to him as the papers appear in the future.

A. Papers Published

1. W. J. Padgett and L. J. Wei, Maximum likelihood estimation of a distribution function with increasing failure rate based on censored observations, Biometrika 67 (1980), 470-474.

2. K. F. Cheng and R. L. Taylor, On the uniform complete convergence of estimates for multivariate density functions and regression curves, Annals, Inst. Statist. Math., 32 (1980), 187-199.

3. L. J. Wei, A generalized Gehan and Gilbert test for paired observations which are subject to arbitrary right censorship, Journal of the American Statistical Association (1980), 634-637.

4. L. J. Wei, Recent development in the design of sequential medical trials, Proc. Conf. on the Recent Developments in Statistics, Taipei, Taiwan, (1980), 349-356.

5. W. J. Padgett and L. J. Wei, A Bayesian nonparametric estimator of survival probability assuming increasing failure rate, Communications in Statistics, Theory & Methods A10 (1981), 46-63.

6. J. O. Howell and R. L. Taylor, Marcinkiewicz-Zygmund type weak laws of large numbers for unconditional random elements in Banach spaces, Proc. Probab. in Banach Spaces, III, LECTURE NOTES IN MATHEMATICS, Vol. 860 (1981), Springer-Verlag, 219-231.

7. J. O. Howell, R. L. Taylor and W. A. Woyczynski, Stability of linear forms in independent random variables in Banach spaces, Proc. Probab. in Banach Spaces, III, LECTURE NOTES IN MATHEMATICS, Vol. 860 (1981), Springer-Verlag, 231-245.

B. Papers Accepted for Publication

8. A.N.V. Rao and W. J. Padgett, On the solution of a class of stochastic integral systems, Journal of Integral Equations (to appear in 1981).

9. L. J. Wei, Estimation of location difference for fragmentary samples, Biometrika (to appear in 1981).

10. L. J. Wei, Asymptotic conservativeness and efficiency of Kruskal-Wallis test for k dependent samples, Journal of the American Statistical Association (to appear in 1981).

11. R. L. Taylor and C. A. Calhoun, On the almost sure convergence of weighted sums of random elements in D[0,1], Int. J. of Math & Math. Sci. (to appear, 1981).

12. W. J. Padgett, Bayes estimators of reliability for the inverse Gaussian model, Statistics T. R. #53, Univ. of S. C., 1980, IEEE Transactions on Reliability (to appear in 1981).

13. W. J. Padgett, Approximate prediction intervals for the mean of future observations from the inverse Gaussian distribution, Statistics T. R. #65, Univ. of S. C., 1980, Journal of Statistical Computation and Simulation (to appear)

14. W. J. Padgett and L. J. Wei, A sequential test and interval estimation in time truncated life testing, Statistics T. R. #49, September, 1979, revised for Sankhyā, tentatively accepted.

15. W. J. Padgett and L. J. Wei, Estimation of the ratio of scale parameters in the two sample problem with arbitrary right censorship, Statistics T. R. #69-R, Univ. of S. C., 1980, under revision for Biometrika.

16. L. J. Wei, Friedman's urn model, Encyclopedia of Statistical Sciences (to appear).

17. L. J. Wei, Interval estimation on location difference with incomplete data, Biometrika (to appear).

C. Papers Submitted for Publication

18. K. Y. Liang and W. J. Padgett, Nonparametric empirical Bayes estimation of reliability, Statistics T. R. #58, Univ. of S. C., 1980, submitted to Metron.

19. W. J. Padgett, On estimation of reliability for the Birnbaum-Saunders fatigue life model, Statistics T. R. #64, Univ. of S. C., 1980, submitted to Journal of Statistical Computation and Simulation.

20. W. J. Padgett and L. J. Wei, Conditionally distribution-free test for censored bivariate observations, Statistics T. R. #41, Univ. of S. C., July, 1979, (submitted for publication).

21. W. J. Padgett and L. J. Wei, Interval estimation after sequential testing based on the total time on test, Statistics T. R. #50, Univ. of S. C., October 1979, submitted to Annals of the Institute of Statistical Mathematics.

22. W. J. Padgett and D. T. McNichols, Estimation under reliability growth assuming gamma failure models, Statistics T. R. #70, Univ. of S. C., 1981, (submitted for publication).

23. R. L. Taylor, Convergence of weighted sums of random elements in type p spaces, Statistics T. R. #43, Univ. of S. C., 1979, (submitted for publication).

24. R. L. Taylor, Complete convergence for weighted sums of arrays of random elements, Statistics T. R. #48, Univ. of S. C., 1979, (submitted for publication.

25. R. L. Taylor, Tightness of measures and laws of large numbers in Banach spaces, Statistics T. R. #35R, Univ. of S. C., 1979, (submitted for publication).

26. R. L. Taylor and P. Z. Daffer, Stochastic convergence of linear forms in D[0,1], Statistics T. R. #72, Univ of S. C., 1981, (submitted for publication).

27. L. J. Wei, Asymptotically distribution-free simultaneous confidence region of treatment differences in a randomized complete block design, revised for the Journal of Royal Statistical Society (B).

28. L. J. Wei, The analysis of a randomized complete block design when observations are subject to arbitrary right censorship, (submitted for publication).

29. L. J. Wei, A conditionally distribution free rank test for testing equality of distribution functions of correlated variables from fragmentary samples, (submitted for publication).

30. E. V. Slud and L. J. Wei, Two-sample repeated significance tests based on the modified Wilcoxon statistic, (submitted for publication).

31. L. J. Wei and M. H. Gail, Nonparametric estimation for a scale-change model with censored observations, (submitted for publication).

5. Professional Personnel Associated with the Research Effort

In addition to the principal investigators, Professors Padgett, Taylor and Wei, D. T. McNichols was a research assistant under the contract and has performed the computing for many examples used in the research papers, performed reference searches, and proofread manuscripts. This student successfully completed the written and oral comprehensive examinations for the Ph.D. program in September, 1979, and began to embark into an area of research for the Ph.D. dissertation in the fall of 1980, expecting to receive the doctorate in the spring of 1982.

Additionally, during the grant period, Professor Taylor has directed three students for their degrees of Master of Science: (i) Connie F. Sawyer, June, 1979, "The Markov Source as a Model for Natural Languages"; (ii) Patrick E. Flanagan, May, 1980, "Computer programs for kernel estimates of a probability density function"; (iii) Carol A. Calhoun, May, 1980, "Laws cf large numbers for D[0,1] and estimation of density functions." Professor Padgett has also directed three students' M.S. theses: (i) Y. S. Lin "Confidence bounds on reliability for the inverse Gaussian model," completed in May, 1980; (ii) K. Wang, "Bayes estimation of reliability for the inverse Gaussian lifetime model", July, 1980; and (iii) C. Hinayon, "Estimation for the Birnbaum-Saunders fatigue life distribution", July, 1980. They also were directing two Ph.D. students and four additional masters' students in 1980-81, and these students will complete their theses during the summer, 1981.

6. Interactions

The principal investigators presented (invited and contributed) papers at various conferences.

(i) W. J. Padgett and L. J. Wei, "The maximum likelihood estimation of a distribution function with monotone failure rate based on censored observations." The Annual Joint Meetings of ASA and IMS in Washington, D.C.

(ii) R. L. Taylor (joint with P. Z. Daffer), "Convergence of weighted sums of random elements in D[0,1]." Annual Meeting of IMS in Washington, D.C.

(iii) L. J. Wei, "A generalized Gehan and Gilbert test for paired observations which are subject to arbitrary right censorship." Annual Meeting of ASA in Washington, D.C.

(iv) L. J. Wei, "The recent development of designs of sequential medical trials" (invited talk). Conference on Recent Development of Statistics at Academia Sinica, Taiwan.

(v) W. J. Padgett (joint with L. J. Wei), "A nonparametric Bayesian estimator of survival probability assuming increasing failure rate". Eastern Regional Meeting of IMS, Charleston, S.C.

(vi) L. J. Wei, "Analysis of randomized complete block design with arbitrary right censorship" (invited talk). Eastern Regional Meeting of IMS, Charleston, S.C.

(vii) W. J. Padgett and Y. S. Lin, "Some confidence bounds on reliability for the inverse Gaussian model". The 2nd Annual meeting of S.C. Chapter of ASA.

(viii) R. L. Taylor, "Complete convergence for weighted sums of arrays of random elements." Eastern Regional Meeting of the Institute of Mathematical Statistics, March 12-14, 1980, Charleston, S.C.

(ix) R. L. Taylor and Carol Calhoun, "Strong laws of large numbers for D[0,1]". The 2nd Annual Meeting of the S.C. Chapter of ASA.

(x) L. J. Wei, "Asymptotically distribution-free simultaneous confidence region of treatment differences in a randomized complete block design". The 2nd Annual Meeting of the S.C. Chapter of ASA.

(xi) W. J. Padgett (joint with L. J. Wei), "A sequential test and interval estimation in time truncated life testing", Joint Statistical Meetings, Houston, Texas, August 10-14, 1980.

(xii) R. L. Taylor, "Marcinkiewicz-Zygmund type weak laws of large numbers for unconditional random elements in Banach spaces," Invited Presentation at the Third International Conference on Probability in Banach Space, Tufts University, Mass., Aug. 8, 1980.

(xiii) W. J. Padgett and D. T. McNichols, "Estimation from development test data assuming gamma lifetimes," The 3rd Annual Meeting of the S.C. Chapter of ASA, April 10, 1981.

(xiv) R. L. Taylor and Carol Calhoun, "Weak laws of large numbers for randomly weighted sums of random variables," The 3rd Annual Meeting of the S.C. Chapter of ASA, April 10, 1981.

(sv) L. J. Wei, Invited talk, Biometrics Spring Meeting, Richmond, Virginia, March, 1981.

(xvi) L. J. Wei, Invited presentation to the Washington, D.C., Chapter of ASA, May, 1981.

In addition, Padgett gave colloquium talks at Auburn University and the University of Florida, and Taylor gave a talk at the University of Central Florida. Wei gave talks at National Cancer Institute, University of Maryland-College Park, University of North Carolina-Chapel Hill, University of Wisconsin-Madison, Ohio State University, and George Washington University.

7. Inventions, Patent Disclosures, and Applications Stemming from the Research Project

No inventions or patents have stemmed from this research.

The results reported in Sections 3 and 4 have wide application in the estimation and assessment of reliability of military equipment both parametrically and nonparametrically. Thus, maintenance policies and development of new more reliable equipment may be formulated using the statistical procedures and theory from these general results. In particular, for reliability and survival analysis, accurate and convenient estimates of survival probabilities, probability density functions, and failure rate functions can be obtained only assuming an increasing failure rate. Also, nonparametric interval estimation and testing can be performed. Hence, assumptions of various parametric failure models are not necessary for estimation or testing for different types of equipment and experimental situations.

8. Other Professional Activities

During this reporting period, the principal investigators have been involved in numerous professional activities which are intimately related to the research efforts on this contract. The principal investigators have refereed a total of twenty-seven manuscripts during June 1, 1979 to May 31, 1981, three for the Journal of Multivariate Analysis, one for Glasnik Matematički, three for IEEE Transactions on Reliability, five for the Journal of the American Statistical Association, three for Biometrics, one for the Journal of Statistical Inference and Planning, three for Communications in Statistics, three for Journal of Controlled Clinical Trials, one for the Canadian Journal of Statistics, one for Technometrics, one for the Journal of Econometrics, one for the Journal of Integral Equations, and one for the International Journal of Mathematics and Mathematical Sciences. In addition, twelve reviews of papers were written for Mathematical Reviews and eight for the Zentralblatt für Mathematik. Also, three NSF proposals were refereed.

W. J. Padgett and R. L. Taylor were the program co-chairman for the Eastern Regional Meeting of the Institute of Mathematical Statistics held on March 12-14, 1980. They were responsible for arranging six invited speaker sessions of the American Statistical Association. R. L. Taylor was chairman of a session on "Reliability and Growth Models" at the 1980 SREB Summer Research Conference on Statistics in Pensacola, Florida, and is program co-chairman for the 1982 SREB Summer Research Conference on Statistics. Also, Taylor was invited to talk at the Third International Conference on Probability in Banach Spaces held August 4-15, 1980, in Boston, Massachusetts. W. J. Padgett has served as an Associate Editor of the Journal of Statistical Computation and Simulation since August, 1980. L. J. Wei has served on the Public Advisory

Committee of NIH and chaired a session at the 1981 Spring Biometric Society Meeting in Richmond, Virginia.